On almost contact metric 2- and 3-hypersurfaces in W_4-manifolds

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The class of W_4-manifolds is one of so-called "small" Gray-Hervella classes [1] of almost Hermitian manifolds. Some specialists identify this class with the class of locally conformal Kählerian (LCK-) manifolds that is not absolutely correct. In fact, the class contains all locally conformal Kählerian manifolds, but coincides with the class of LCK-manifolds only for dimension at least six [2]. W_4-manifolds were studied in detail by such outstanding mathematicians as A. Gray (USA), V.F. Kirichenko (Russian Federation) and I. Vaisman (Israel).

As it is known, almost contact metric structures are induced on oriented hypersurfaces of an almost Hermitian manifold. We remind that the almost contact metric structure on an odd-dimensional manifold N is defined by the system of tensor fields $\{\Phi,\, \xi,\, \eta,\, g\}$ on this manifold, where ξ is a vector field, η is a covector field, Φ is a tensor of the type (1, 1) and $g = \langle\cdot,\cdot\rangle$ is a Riemannian metric [5], [6]. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \Phi(\xi) = 0,\ \eta\circ\Phi = 0,\ \Phi^2 = -id + \xi\otimes\eta,$$

$$\langle \Phi X, \Phi Y\rangle = \langle X, Y\rangle - \eta\,(X)\,\eta\,(Y)\,,\ X, Y \in \aleph(N),$$

where $\aleph(N)$ is the module of the smooth vector fields on N.

In [4] it has been proved that if the almost Hermitian manifold is of class W_4 and the type number of its hypersurface is equal to one, then the almost contact metric structure on such a hypersurface is identical the structure on totally geodesic hypersurface. The similar results were obtained for 0- and 1-hypersurfaces of W_1- and W_3-manifolds [5], [6].

The main result of the present communication is the following:

Theorem. *3-hypersurfaces of W_4-manifolds admit an almost contact metric structure that can be identical to the structure induced on 2-hypersurfaces of such manifolds.*

Taking into account the above mentioned fact that the class of W_4-manifolds contains all LCK-manifolds, we get the following:

Corollary. *3-hypersurfaces of locally conformal Kählerian manifolds admit an almost contact metric structure that can be identical to the structure induced on 2-hypersurfaces of such manifolds.*

We remark that the structure induced on 2- and 3-hypersurfaces of W_4-manifolds does not belong to any well-studied classes of almost contact metric structures (cosymplectic, nearly cosymplectic, Kenmotsu, Sasaki structures etc).

Bibliography

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