About the generalized symmetry of geometric figures weighted regularly and easily by "physical" scalar tasks

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Let us have geometrical figure F with discrete group of symmetry G and finite set $N = \{1, 2, ..., m\}$ of "indexes", which mean a non-geometrical feature. On fix a certain transitive group P of permutations over N. We will note with the symbol F_i the intersection of geometric figure F with the fundamental domain S_i of the group G. Ascribe to each interior point M of F_i the same "index" r from the set N . We obtain one figure $F^{(N)}$, weighted regularly and easily with summary load N.
Let each "index" r from the set N have a scalar nature (temperature, density, color). The mixed transformation $\tilde{g}$ of the "indexed" figure $F^{(N)}$ is composed of two independent components: $\tilde{g} = gw$, where g is pure geometrical isometric transformation and w is certain complex rule which describes the transformation of the "indexes". If the rule w is the same for every "indexed" point of $F^{(N)}$, then the mixed transformation $\tilde{g}$ is exactly a transformation of Zamorzaev's P-symmetry. The set of transformations of P-symmetry of "indexed" figure $F^{(N)}$ forms a minor or semi-minor group of P-symmetry , where is subgroup of the direct product of the group P with generating group G [1,3].
The "indexes" r_i and r_j, ascribed to the points which belong to distinct domains F_i and F_j, are transformed, in general, by different permutations p_i and p_j from group P. In this case the rule w is composed exactly from $|G|$ components-permutations $p \in P$. In conditions of this case the transformation $\tilde{g} = gw$ is exactly a transformation of W_p-symmetry [2-5]. The set of transformations of W_p-symmetry of the given "indexed" figure $F^{(N)}$ forms a semi-minor or pseudo-minor group of W_p-symmetry, where is subgroup of the left standard Cartaisian wreath product of groups P and G.

Bibliography

[1] A.M. Zamorzaev. *On the groups of quasi-symmetry (P-symmetry).* (Russian). Soviet. Phys. Crystallogr. vol. 12, no 5 (1967), pp. 819-825.

[2] V.A. Koptsik and I.N. Kotsev, *To the theory and classification of color symmetry Groups. II. W-symmetry.* (Russian). OIYI Reports, R4-8068, Dubna, 1974.

[3] A.M. Zamorzaev, Yu.S. Karpova, A.P. Lungu and A.F. Palistrant. *P-symmetry and its further development.* (Russian). Chisinau: Shtiintsa, 1986.

[4] A.P. Lungu, *Classification of groups of W-symmetry*, (Russian). Studies in modern algebra and geometry. p. 79-84, Chisinau: Shtiintsa, 1983.

[5] Alexandru Lungu, *Cristallografic punctual groups of cyclical W_p-symmetries*, (Romanian). Studia Universitatis Moldaviae, Seria: Shtiintse exacte shi economice, nr.2(102) (2017), pp.3-12.

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