On weak expressibility of formulas in the simplest non-trivial propositional provability logic

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We consider the simplest non-trivial extension of the propositional provability logic, denoted by $G4$, which is based on variables and logical connectives $\&, \vee, \supset, \neg, \Delta$, axioms of the classical logic of propositions and Δ-axioms:

$$\Delta(p \supset q) \supset (\Delta p \supset \Delta q),\ \Delta p \supset \Delta\Delta p,$$

$$\Delta(\Delta p \supset p) \supset \Delta p,\ \Delta(p \supset p) \supset (p \supset p).$$

$$\Delta\Delta p,\ (\Box p \supset \Box q) \vee (\Box q \supset \Box p),$$

where $\Box p$ denotes $(p\&\Delta p)$. Rules of inference of $G4$ are the rules of the classical logic of propositions and the rule $\frac{A}{\Delta A}$.
They say formula F is weak-expressible by formulas of the system Σ in the logic L if F can be obtained from unary formulas of L and from formulas of Σ by applying the rule of weak substitution (which allows to pass from the formulas A and B to the result of the substitution of one of them in another one instead of all occurrences of the same variable, say p) and by the rule of replacement by an equivalent formula (which permit to pass from the formula A to an equivalent to it in L formula B).
They say the system of formulas Σ is complete with respect to weak-expressibility in the logic L if any formula of the calculus of L is weak-expressible via Σ in L.
In the present paper we found out the conditions for a system of formulas Σ to be complete as to weak-expressibility in the logic $G4$. **Acknowledgement.** *The work was partially supported by the research projects 18.50.07.10A/PS and 15.817.06.13A.*